STAT 113 – Review Problems

Testing a Population mean: the z-test (9.6.)

1) A manager of public health services in an area downwind of a nuclear test site wants to test the hypothesis that the mean amount of radiation in the form of Strontium-90 in the bone marrow (measured in picocuries) for citizens who live downwind of the site does exceed that of citizens who live upwind from the site. It is known that "upwinders" have a mean level of Strontium-90 of 1 picocurie. Measurements of Strontium-90 radiation for a sample of $n = 16$ citizens who live downwind of the site was taken, giving $\bar{X} = 3$. The population standard deviation is $\sigma = 4$.

a) Test the (research, alternative) hypothesis that downwinders have a higher Strontium-90 level than upwinders. Assume normality and use a significance level of $\alpha = 0.05$.
(i) State H_0 and H_1
(ii) State the appropriate test statistic
(iii) Determine the critical region of the test
(iv) State your decision
b) What would constitute a type-II error in this setup? *Describe in less than 20 words.*
c) Find the power of the test for $\mu = 1.2$.
d) How large a sample size is required for the power at $\mu = 1.2$ to be at least 0.9?

Estimation of a Population mean (8.5)

e) Find a 95% confidence interval for μ.

Testing a Population mean: the z-test (9.6.)

2) Suppose $\bar{X}$ is the mean of a normal random sample of size 25, where $\sigma = 10$. Test $H_0 : \mu = 75$ versus $H_1 : \mu < 75$ by rejecting the null hypothesis when $\bar{X} \leq 71.08$.
(a) Find α.
(b) Find β for $\mu = 73$.

Testing a Population mean: the t-test (9.6)

3) Test $H_0 : \mu = 15$ versus $H_1 : \mu \neq 15$ if a normal random sample of size 20 gives $\bar{X} = 9.6$ and $s = 4.7$. Let $\alpha = 0.05$.

Testing the difference between two population means (9.9)

4) Per captia income is defined as the annual total personal income of the residents of a region divided by the resident population of that region as of July 1 of that year. To compare eastern and western states we record annual total personal income for the 17 states west of the Mississippi, and the 31 states east of the Mississippi `Please check numbers`. For the 17 states west of the Missisippi, the sample mean is $\bar{x} = 17001$, and the sample **standard deviation** is $S_x = 2264$. For the 31 states east of the Missisippi the sample mean is $\bar{y} = 18431$, and $S_Y = 3095$.
Are incomes in the west and the east different?
a) What would be the approrpriate test statistic to formally test the hypothesis:

$$H_0 : \mu_x = \mu_y \text{ vs. } H_1 : \mu_x \neq \mu_y$$

State the test statistic (formula). **Don't actually do the test!**

Estimation of the difference between two population means (8.6)

b) Give a 95% confidence interval for $\mu_1 - \mu_2$.