STARTED: | FINISHED:

Name: Section:

STAT 113, Spring 99 – Quiz 1

1. Consider a standard normal random variable $Z \sim N(0,1)$.

Find the following probabilities *(Note: This is like homework problem 5.24)*

1a.[1pt] $P(.7 < Z < 1.3)$

$P(.7 < Z < 1.3) = P(0 < Z < 1.3) - P(0 < Z < .7) = 0.4032 - 0.2580 = .15$

1b.[1pt] $P(Z < -1.0)$

$P(Z < -1.0) = 0.5 - P(0 < Z < 1.0) = 0.16$

Find the value c such that *(Note: This is like homework problem 5.25)*

1c.[1pt] $P(Z > c) = 0.3$

For $c = 0.52$ we find $P(0 < Z < c) = 0.2$, i.e. $P(Z > c) = 0.3$

1d.[1pt] $P(Z > c) = 0.9$

For $c = 1.28$ we find $P(0 < Z < c) = 0.4$, i.e. $P(Z > -1.28) = P(-1.28 < Z < 0) + P(0 < Z) = 0.4 + 0.5 = 0.9$

2. Let c be a constant and consider the density function

$$f(y) = \begin{cases} cy^2 & \text{if } 0 \le y \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Note: This problem is (almost) identical to the homework problem 5.1.

2a.[1pt] Find the value of c.

$$1.0 = c\int_0^3 y^2 dy = c\left.\frac{y^3}{3}\right|_0^3 = c\,9 \quad \Rightarrow c = 1/9.$$

Partial Credit: [1/2pt] for correct integral expression

2b.[1pt] Find the cumulative distribution function $F(y)$.

$$F(y) = P(Y \le y) = c\int_0^y t^2 dt = c\,\frac{y^3}{3} = y^3/27.$$

Partial Credit: [1/2pt] for $F(y) = P(Y \le y)$

2c.[1pt] Compute $P(1 \le y \le 1.5)$.

$= F(1.5) - F(1) = (1.5^3 - 1)/27 = 0.09$

2d.[1pt] Find $E(Y)$.

$$E(y) = c\int_0^3 y\,y^2 dy = c\left.\frac{y^4}{4}\right|_0^3 = 3^4/36 = 2.25$$

Partial Credit: [1/2pt] for $E(y) = c\int_0^3 y\,y^2 dy$.

3. [4pts] A random sample of $n = 747$ obituaries published in Salt Lake Citiy newspapers revealed that 46% (i.e.,$X = 344$) of the decedents died within the three-month period following their birthdays.

Find the probability that 46% or more would die in that interval if deaths occured randomly throughout the year.
What would you conclude on the basis of your answer?
Note: This is like homework problems 7.29 and 7.32 – just with different numbers.

Assuming that deaths orrur randomly throughout the year X is a binomial r.v. with $n = 747$ and $p = 3/12 = 0.25$, i.e., $X \sim Bin(n, p)$.
Partial Credit: [1pt] for stating Binomial, [1pt] for correct p and n.

$$P(X \geq 344) = P\left(\frac{X - np}{\sqrt{n\,p\,(1-p)}} \geq \frac{344 - np}{\sqrt{n\,p\,(1-p)}}\right) \approx P(Z \geq 13.28) = 0.0000.$$

Partial Credit: [1pt] for using normal approximation; [1pt] for correct z-transformation.

4. Assume X is a normal random variable with unknown mean μ and standard deviation $\sigma = 2.0$ (i.e., variance $\sigma^2 = 4.0$).

4a.[2pt] Find a cutoff c such that $P(-c \leq \frac{X-\mu}{\sigma} \leq +c) = 0.95$

Note that $Z = (X - \mu)/\sigma$ is standard normal $N(0, 1)$. Find from the normal table $P(0 < Z < 1.96) = 0.4750$, i.e. $P(-1.96 < Z < 1.96) = 0.95$.
[1pt] for stating that $Z = (X - \mu)/\sigma$ is $N(0, 1)$.

4b*.[2pt] For appropriate coefficients a_1, a_2 and b_1, b_2, the following statement is correct:

$$P(a_1 + a_2\, X \leq \mu \leq b_1 + b_2\, X) = 0.95 \tag{1}$$

Find a_1, a_2, b_1, and b_2.

$$P(-c \leq \frac{X-\mu}{\sigma} \leq +c) = 0.95 \quad \Rightarrow P(X - c\sigma < \mu < X + c\sigma) = 0.95$$

i.e. $a_1 = -c\,\sigma$, $a_2 = 1.0$, $b_1 = c\,\sigma$, and $b_2 = 1.0$.