MIDTERM EXAM

STA 214
3/12/1998.

Name ______________________________

ID number ______________________________

Notes:

1. This is an open book and open notes exam.
2. You must show your work and/or explain your answer in order to receive credit.
3. The exam has 4 problems and carries 100 points.

Problem	1	2	3	4	Total
Score					

1. (Markov Chain). The first column of a transition matrix $\mathbb{P} = (p_{i,j}), i = 0, 1, 2, \ldots;\ j = 0, 1, 2, \ldots$ is $\{1/3,\ 1/3,\ 1/3,\ 1/3,\ \ldots\}$ and $p_{i,i+1} = 2/3$.

(a) Draw the graph of the MC defined by $\mathbb{P}$.

(b) Find the stationary distribution, if any.

2. Let

$$X_t - \frac{1}{2}X_{t-1} = Z_t, t \in T,$$

be an $AR(1)$ process.

(a) Find its autocovariance function, $\gamma(h)$. [Hint: Use the $MA(\infty)$ representation of X_t.]

(b) Find the spectral density of X_t, $f(\omega)$.

3. Brownian motion $B_t, t \geq 0$ is observed at two time instances: $t = 0.3$ and $t = 0.7$.
(a) Find the coefficient of correlation between the observed values, $B_{0.3}$ and $B_{0.7}$.

(b) Do the same as in (a), for the Brownian Bridge.

4. What is a *long memory process*? What is the *Hurst* exponent?