Homework 1 for STA 250 – Fall 2017

Due in class September 6, 2017 (Show your work!)

Note: This homework will be graded based only on participation and will count for 1% of the course grade. It is mainly for you (and me) to get a sense of whether you have the necessary background for this course.

1. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer and $1-p$ the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly?

2. A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 p.m., find the probability that the first to arrive has to wait longer than 10 mins.

3. Let Z be a standard normal random variable, and for a fixed x, set

$$X = \begin{cases} Z & \text{if } Z > x \\ 0 & \text{otherwise.} \end{cases}.$$

   Show that $E[X] = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

4. Let U be a **Uniform**$(0,1)$ random variable, and suppose that the conditional distribution of X, given that $U = p$, is **Binomial**(n,p). Find the probability mass function of X.

5. If X and Y are indepenent Poisson random variables with respective parameters λ_1 and λ_2, what is the distribution of $X+Y$?

6. If the distribution of X is **Uniform**$(0,1)$ and given $X = x$, the distribution of Y is **Uniform**$(0,x)$. Find $E[Y]$ and $\text{Var}[Y]$.