Covariance models for hierarchical data

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Fixed occasion longitudinal data

Data measured at regular time intervals on each subject.

- $y_{t,j}$ = observation on subject j at time t.
- $j = 1, \dots, m$;
- $t = 1, \dots, T$.

$$\begin{pmatrix} y_{1,1} & y_{2,1} & \cdots & y_{T,1} \\ y_{1,2} & y_{2,2} & \cdots & y_{T,2} \\ \vdots & & & \vdots \\ y_{1,m} & y_{2,m} & \cdots & y_{T,m} \end{pmatrix}$$

Fixed occasion longitudinal data

Structurally, this can be thought of as two-level hierarchical data:

- A subject is a *group*, as we have multiple observations within a subject.
- $t = 1, \ldots, T$ indexes the multiple observations within a group (subject).

One goal of HM is to *account for correlation of observations within a group* .

For longitudinal data, this translates to the goal of

accounting for temporal correlation within a subject.

The difference: There is some structure to the "units" within a group:

- time is an ordered quantity;
- there might be some similarity of "units" *across* groups, as they correspond to common times.

Longitudinal data - an alternative viewpoint

Alternatively, you could structure the data as

- $y_{i,t}$ = observation on subject i at time t.
- $i = 1, \ldots, n$
- $j = 1, \ldots, T$.

You could think of each time point as being a group, within which there are multiple observations (subjects).

Problem with this perspective: In most applications, we are

- concerned with temporal dependence *within a subject*,
- not dependence of subjects *within a timepoint*.

However, there are some situations when this perspective is reasonable:

- if time can be viewed as a "sampled" qunatity without an ordered effect.
- if the similarity due to common time session dominates similarity due to other factors.

Example: Sleep deprivation study

$y_{t,j}$ = reaction time of subject j, t days after beginning of study.

$t = 0, \ldots, 9$.

$j = 1, \ldots, 18$.

```
sleep[1:15,]
```

```
##    Reaction Days Subject
## 1  249.5600    0     308
## 2  258.7047    1     308
## 3  250.8006    2     308
## 4  321.4398    3     308
## 5  356.8519    4     308
## 6  414.6901    5     308
## 7  382.2038    6     308
## 8  290.1486    7     308
## 9  430.5853    8     308
## 10 466.3535    9     308
## 11 222.7339    0     309
## 12 205.2658    1     309
## 13 202.9778    2     309
## 14 204.7070    3     309
## 15 207.7161    4     309
```

Example: Life satisfaction

$y_{t,j}$ = life satisfaction of j at age t.

$t = 55, 56, \ldots, 59, 60.$

$j = 1, \ldots, 1237.$

```
happy[1:10,]

##      id    j gender age married nrchildren yearsEd emplStat totIncBeforeG
## 1   101 1985      1  55       1          0    15.0        1      23219.03
## 2   101 1986      1  56       1          0    15.0        1      23263.78
## 3   101 1987      1  57       1          0    15.0        1      23273.50
## 4   101 1988      1  58       1          0    15.0        1      23559.82
## 5   101 1989      1  59       1          0    15.0        1      23110.63
## 6   901 2006      2  55       2          0    10.5        1      22501.25
## 7  1701 2003      1  55       1          0    15.0        1      68116.78
## 8  1701 2004      1  56       1          0    15.0        1      78862.02
## 9  1701 2005      1  57       1          0    15.0        1      77910.88
## 10 1701 2006      1  58       1          0    15.0        1      79684.43
##    totIncAfterG laborInc state healthSat lifeSat emplStat55
## 1      16164.22 23060.29     0         8       8          1
## 2      16072.46 23263.78     0         7      10          1
## 3      16108.25 23273.50     0         5       8          1
## 4      16268.29 23559.82     0         8       8          1
## 5      16110.02 22857.30     0         7       8          1
## 6      14876.25 22396.00     0         7       6          1
## 7      45437.78 67975.36     0         9       8          1
## 8      52091.36 78720.60     0         9      10          1
## 9      50958.88 77769.45     0         9      10          1
## 10     52345.42 79543.00     0        10       9          1
```

Example: Life satisfaction

Notice:

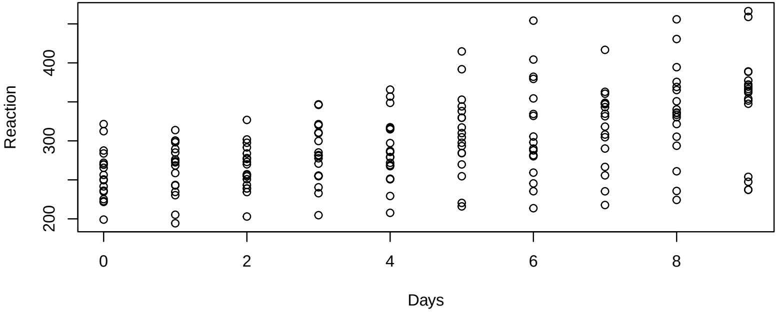
- The “times” are not actually at the same time.
- Subjects have varying amounts of data in the study.
- All data corresponds to one of the six ages (fixed occasion data).

Model fitting, diagnostics and model building

We will first develop some methods by analyzing the `sleep` data.

Questions:

- What are the effects of sleep deprivation on reaction time, on average?
- How do these effects vary across people?



Model fitting, diagnostics and model building

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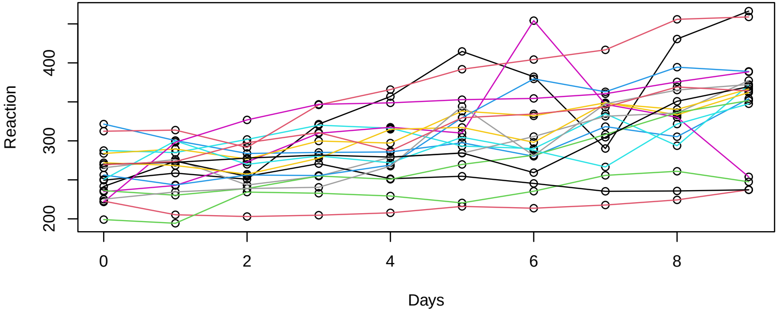


Start simple, then criticize

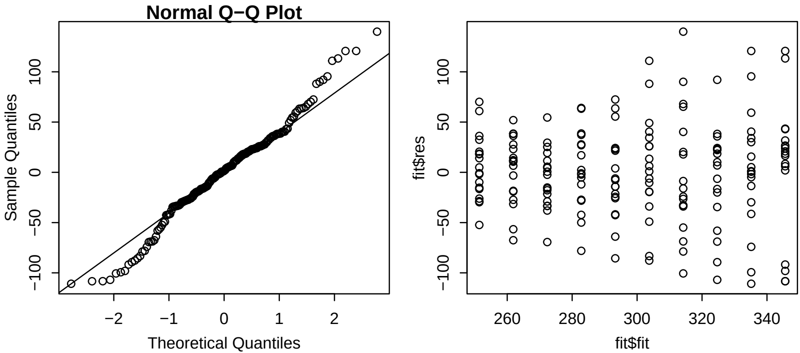
Time as a categorical predictor:

```
fit_fact<-lm( Reaction~as.factor(Days), data=sleep)

BIC(fit_fact)

## [1] 1955.84
```

Time as a continuous predictor:

```
fit<-lm( Reaction~Days, data=sleep)

BIC(fit)

## [1] 1915.872
```

```
summary(fit)$coef

##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 251.40510   6.610154 38.033169 2.156888e-87
## Days         10.46729   1.238195  8.453663 9.894096e-15
```

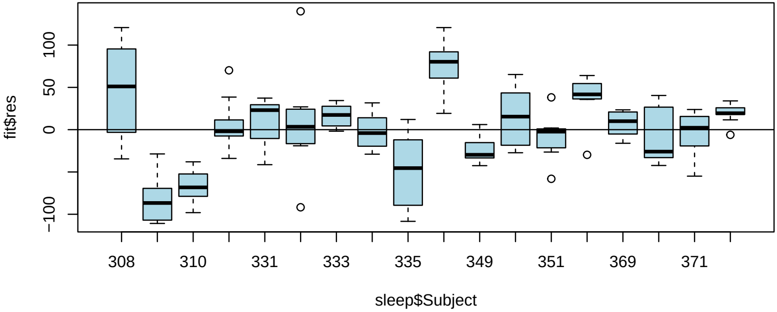


Recall: General order of importance of modeling assumptions:

1. independence;
2. constant variance;
3. normality.

Checking independence

```
plot(fit$res~sleep$Subject,col="lightblue") ; abline(h=0)
```



There is clear evidence that the observations are not independent.

- many subjects have either all resids above zero, or all resids below.

Compound symmetry/exchangeable covariance model

Q: How can we model dependence within a subjct?

A1: You already have one tool at your disposal, the HNM:

Random effects representation:

$$\begin{aligned} y_{t,j} &= \beta_0 + \beta_1 \times t + b_j + \epsilon_{t,j} \\ \{b_j\} &\sim \ iid \ N(0, \tau^2) \\ \{\epsilon_{t,j}\} &\sim \ iid \ N(0, \sigma^2) \end{aligned}$$

Correlated data representation:

$$y_{t,j} = \beta_0 + \beta_1 \times t + e_{t,j}$$

$$\text{Cov}\begin{pmatrix} e_{1,j} \\ e_{2,j} \\ \vdots \\ e_{T,j} \end{pmatrix} = \begin{pmatrix} \tau^2 + \sigma^2 & \tau^2 & \cdots & \tau^2 \\ \tau^2 & \tau^2 + \sigma^2 & \cdots & \tau^2 \\ \vdots & & & \vdots \\ \tau^2 & \cdots & \tau^2 & \tau^2 + \sigma^2 \end{pmatrix}$$

Recall, under this model $\text{Cor}[y_{t_1,j}, y_{t_2,j}] = \frac{\tau^2}{\tau^2+\sigma^2}$.

Compound symmetry/exchangeable covariance model

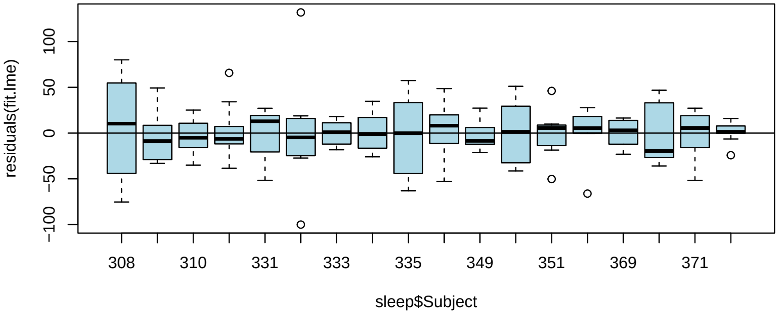
```
fit.lm<-lm(Reaction~Days , data=sleep)
fit.lme<-lmer(Reaction~Days+(1|Subject) , data=sleep)

BIC(fit.lm)

## [1] 1915.872

BIC(fit.lme)

## [1] 1807.237
```

```
summary(fit.lme)$coef

##              Estimate Std. Error  t value
## (Intercept) 251.40510  9.7467163 25.79383
## Days         10.46729  0.8042214 13.01543
```

Checking residuals

$$\hat{\epsilon}_{t,j} = y_{t,j} - (\hat{\beta}_0 + \hat{\beta}_1 \times t + \hat{b}_j)$$

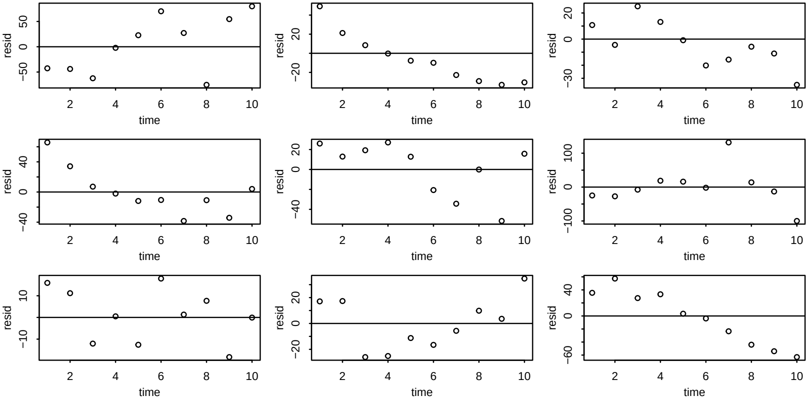
```
plot( residuals(fit.lme)~ sleep$Subject,col="lightblue") ; abline(h=0)
```



Looks much better!

Closer inspection of residuals

$$\hat{\epsilon}_{t,j} = y_{t,j} - (\hat{\beta}_0 + \hat{\beta}_1 \times t + \hat{b}_j)$$



What kind of model might fit better?

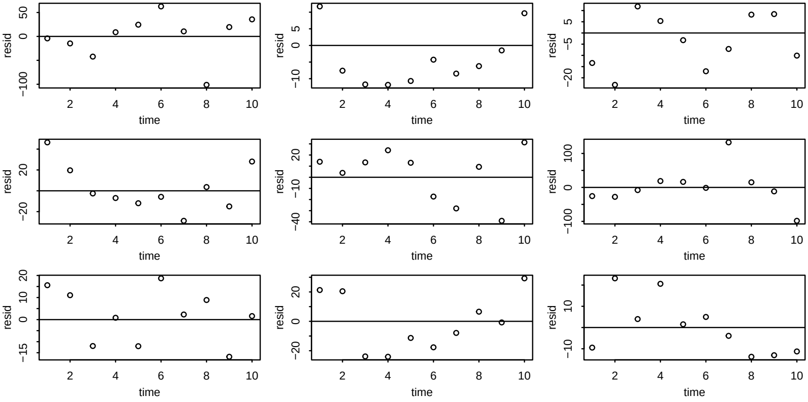
Group-specific time trends

```
fit.lme2<-lmer(Reaction~Days+(Days|Subject) , data=sleep)

BIC(fit.lme)

## [1] 1807.237

BIC(fit.lme2)

## [1] 1774.786
```

```
summary(fit.lme2)$coef

##              Estimate Std. Error   t value
## (Intercept) 251.40510   6.824597 36.838090
## Days         10.46729   1.545790  6.771481
```

Residual analysis

$$\hat{\epsilon}_{t,j} = y_{t,j} - (\hat{\beta}_0 + \hat{\beta}_1 \times t + \hat{b}_{0j} + \hat{b}_{1j} \times t)$$



Better still, but there still seems to be residual dependence.

Residual correlation

```
RES<- tapply(residuals(fit.lme2),list(sleep$Subject,sleep$Days),"c")
round(RES,2)

##          0      1      2      3      4      5      6       7      8      9
## 308  -4.10 -14.63 -42.20   8.78  24.52  62.70  10.54 -101.18  19.59  35.69
## 309  11.73  -7.59 -11.72 -11.84 -10.68  -4.28  -8.46   -6.21  -1.49   9.68
## 310 -13.39 -23.13  11.84   5.34  -3.21 -17.08  -7.13    8.18   8.42 -10.10
## 330  46.45  19.65  -2.55  -6.92 -11.91  -5.77 -28.77    3.60 -14.97  28.08
## 331  13.94   3.94  13.36  24.26  13.02 -17.33 -27.97    9.37 -39.10  31.34
## 332 -25.58 -27.83  -7.87  18.74  16.24  -1.42 132.55   15.02 -11.71 -98.34
## 333  15.60  11.07 -11.96   0.83 -12.05  18.70   2.32    8.89 -16.83   1.60
## 334  21.30  20.49 -23.89 -24.13 -11.32 -17.69  -7.90    6.56  -0.76  29.25
## 335  -9.46  23.16   3.99  20.59   1.52   4.99  -3.91  -13.77 -13.04 -11.26
## 337  26.07   8.41 -32.88   2.54   3.05  10.07   3.39   -3.27  16.80   0.76
## 349   9.91  -7.52 -10.55  -6.20 -22.05 -14.62 -14.47    0.42  16.96  20.68
## 350  17.96 -11.96 -16.29 -34.05 -37.74   5.98  38.62    5.01  19.50  -3.02
## 351  -5.46  36.62  -0.99   2.25 -13.96  11.39 -12.95  -41.55   5.94  24.51
## 352 -50.59  11.92  26.60  32.58  20.46  10.54  -1.86   -9.86  -8.65  -9.76
## 369  17.24   2.42 -20.12 -11.04  14.78   5.84 -24.58   14.07  -5.12   9.78
## 370  -0.53  -6.56 -17.47 -31.19 -19.41  41.95 -36.38   14.76  17.05   8.83
## 371  17.67  10.75   6.73   1.14 -10.96 -15.10 -49.82  -13.94  22.74  31.94
## 372   5.69  -2.00  10.37  11.66 -23.55   7.13   0.25   -2.76  11.41  -5.36
```

Residual covariance

```r
round(cov(RES),1)
```

```
##         0      1      2      3      4      5       6      7      8       9
## 0  468.4   85.5 -157.8 -222.2 -165.6  -97.3  -326.3  107.2   16.7   343.9
## 1   85.5  285.9   50.0   21.9  -28.3  -39.3  -329.3  -42.1  -69.1   243.9
## 2 -157.8   50.0  305.2  176.1   -0.1 -180.4  -100.6  147.7 -115.1   -81.9
## 3 -222.2   21.9  176.1  337.8  209.2  -23.2   136.7 -115.4 -141.8  -163.6
## 4 -165.6  -28.3   -0.1  209.2  293.8   76.0   134.5 -147.3 -116.4  -104.8
## 5  -97.3  -39.3 -180.4  -23.2   76.0  442.2    59.6 -344.9  106.8    34.1
## 6 -326.3 -329.3 -100.6  136.7  134.5   59.6  1512.7   53.1  -53.9 -1012.4
## 7  107.2  -42.1  147.7 -115.4 -147.3 -344.9    53.1  754.5 -152.8  -294.7
## 8   16.7  -69.1 -115.1 -141.8 -116.4  106.8   -53.9 -152.8  281.5    80.2
## 9  343.9  243.9  -81.9 -163.6 -104.8   34.1 -1012.4 -294.7   80.2   926.9
```

Residual correlation

```r
round(cor(RES),2)
```

```
##       0     1     2     3     4     5     6     7     8     9
## 0  1.00  0.23 -0.42 -0.56 -0.45 -0.21 -0.39  0.18  0.05  0.52
## 1  0.23  1.00  0.17  0.07 -0.10 -0.11 -0.50 -0.09 -0.24  0.47
## 2 -0.42  0.17  1.00  0.55  0.00 -0.49 -0.15  0.31 -0.39 -0.15
## 3 -0.56  0.07  0.55  1.00  0.66 -0.06  0.19 -0.23 -0.46 -0.29
## 4 -0.45 -0.10  0.00  0.66  1.00  0.21  0.20 -0.31 -0.40 -0.20
## 5 -0.21 -0.11 -0.49 -0.06  0.21  1.00  0.07 -0.60  0.30  0.05
## 6 -0.39 -0.50 -0.15  0.19  0.20  0.07  1.00  0.05 -0.08 -0.86
## 7  0.18 -0.09  0.31 -0.23 -0.31 -0.60  0.05  1.00 -0.33 -0.35
## 8  0.05 -0.24 -0.39 -0.46 -0.40  0.30 -0.08 -0.33  1.00  0.16
## 9  0.52  0.47 -0.15 -0.29 -0.20  0.05 -0.86 -0.35  0.16  1.00
```

Covariance models

Each model we've considered so far corresponds to a different covariance model:

Linear model: $y_{t,j} = \beta_1 + \beta_2 \times t + \epsilon_{t,j}$

$$\text{Cov}[y_{s,j}, y_{t,j}] = 0$$

HLM with random intercepts: $y_{t,j} = \beta_1 + \beta_2 \times t + b_j + \epsilon_{t,j}$

$$\text{Cov}[y_{s,j}, y_{t,j}] = \tau^2 \ , \quad \text{where } \text{Var}[b_j] = \tau^2$$

HLM with random intercepts and time trends:

$$\text{Cov}[y_{s,j}, y_{t,j}] = \tau_1^2 + \tau_{12} \times (t + s + t \times s \times \tau_1^2) \ , \quad \text{where } \text{Cov}\begin{pmatrix} b_{0,j} \\ b_{1,j} \end{pmatrix} = \begin{pmatrix} \tau_1^2 & \tau_{12} \\ \tau_{12} & \tau_2^2 \end{pmatrix}$$

Residual covariance models

In all the models considered so far, we've assumed

$$\text{Cov}\begin{pmatrix}\epsilon_{1,j}\\ \vdots \\ \epsilon_{n,j}\end{pmatrix} = \sigma^2 \mathbf{I} = \begin{pmatrix}\sigma^2 & 0 & \cdots & 0\\ 0 & \sigma^2 & \cdots & 0\\ \vdots & & & \vdots\\ 0 & \cdots & 0 & \sigma^2\end{pmatrix}$$

- "micro" level errors are independent, or equivalently
- all within-group dependence is explained by random effects.

In cases where it appears random effects are not accounting for the dependence, we may want to allow for non-independence among the $\epsilon_{i,j}$'s.

$$\text{Cov}\begin{pmatrix}\epsilon_{1,j}\\ \vdots \\ \epsilon_{n,j}\end{pmatrix} = \Sigma_\epsilon = \begin{pmatrix}\sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,T}\\ \sigma_{1,2} & \sigma_2^2 & \cdots & \sigma_{2,T}\\ \vdots & & & \vdots\\ \sigma_{1,T} & \cdots & \sigma_{T-1,T} & \sigma_T^2\end{pmatrix}$$

Model fitting with nlme

```
fit.lme4<-lmer(Reaction~Days+(Days|Subject), data=sleep,REML=FALSE)

logLik(fit.lme4)

## 'log Lik.' -875.9697 (df=6)
```

```
library(nlme)

fit.nlme<-lme(Reaction~Days, random=~Days|Subject, data=sleep,method="ML")

logLik(fit.nlme)

## 'log Lik.' -875.9697 (df=6)
```

nlme output

```
summary(fit.nlme)

## Linear mixed-effects model fit by maximum likelihood
##   Data: sleep
##        AIC      BIC    logLik
##   1763.939 1783.097 -875.9697
##
## Random effects:
##  Formula: ~Days | Subject
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 23.780376 (Intr)
## Days         5.716807 0.081
## Residual    25.591842
##
## Fixed effects:  Reaction ~ Days
##                 Value Std.Error  DF  t-value p-value
## (Intercept) 251.40510  6.669396 161 37.69533       0
## Days         10.46729  1.510647 161  6.92901       0
##  Correlation:
##      (Intr)
## Days -0.138
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.94156355 -0.46559311  0.02894656  0.46361051  5.17933587
##
## Number of Observations: 180
## Number of Groups: 18
```

Correlation models

```
?corClasses
```

```
Correlation Structure Classes

Description:

     Standard classes of correlation structures (‘corStruct’) available
     in the ‘nlme’ package.

Value:

     Available standard classes:

  corAR1: autoregressive process of order 1.

 corARMA: autoregressive moving average process, with arbitrary orders
          for the autoregressive and moving average components.

 corCAR1: continuous autoregressive process (AR(1) process for a
          continuous time covariate).
          .
          .
          .
corRatio: Rational quadratics spatial correlation.

corSpher: spherical spatial correlation.

corSymm: general correlation matrix, with no additional structure.
```

Correlation models for longitudinal data

Several of the correlation models are designed specifically for longitudinal data.

- `corAR1`: First-order discrete-time (fixed occasion) autocorrelation model.
- `corARMA`: A general class of discrete-time models.
- `corCAR1`: A first order continuous time model.

In particular, AR1 model is a two-parameter covariance model for which

$$\mathrm{Cov}\begin{pmatrix}\epsilon_{1,j}\\ \vdots \\ \epsilon_{T,j}\end{pmatrix} = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \vdots & & & & \vdots \\ \rho^{T-1} & \rho^{T-2} & \cdots & \rho & 1 \end{pmatrix},$$

where $\sigma^2 > 0$ and $\rho \in (-1, 1)$.

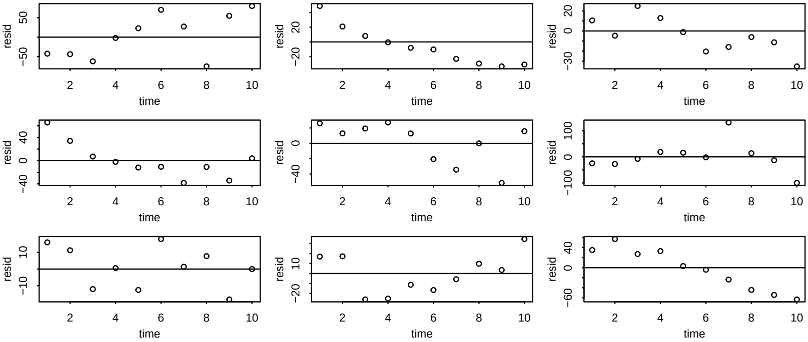
Exercise: Describe the correlation between two observations as a function of their time difference.

Model comparison and selection

Random intercept:

```
fit1<-lme(Reaction~Days, random=~1|Subject, data=sleep,method="ML")
```

$$\hat{\epsilon}_{t,j} = y_{t,j} - (\hat{\beta}_0 + \hat{\beta}_1 \times t + \hat{b}_j)$$



What kind of model might fit better? Random time trend or AR1?

Model comparison

Random intercept and time trend:

```
fit2a<-lme(Reaction~Days, random=~Days|Subject, data=sleep,method="ML")

BIC(fit2a)

## [1] 1783.097
```

Random intercept and autocorrelated errors:

```
fit2b<-lme(Reaction~Days, random=~1|Subject,correlation=corAR1(), data=sleep,method="ML")

BIC(fit2b)

## [1] 1770.795
```

Limitations

```
fit2b<-lme(Reaction~Days, random=~Days|Subject,correlation=corAR1(), data=sleep,method="ML"

## Error in lme.formula(Reaction ~ Days, random = ~Days | Subject, correlation =
corAR1(), :  nlminb problem, convergence error code = 1
##   message = iteration limit reached without convergence (10)
```

Longitudinal data with covariates

Male subset of SOEP data

```
mappy[1:10,c(1,14,4,16,17,7) ]

##      id lifeSat age birthyear logincome yearsEd
## 1   101       8  55      1930  10.04591      15
## 2   101      10  56      1930  10.05470      15
## 3   101       8  57      1930  10.05511      15
## 4   101       8  58      1930  10.06734      15
## 5   101       8  59      1930  10.03707      15
## 7  1701       8  55      1948  11.12692      15
## 8  1701      10  56      1948  11.27367      15
## 9  1701      10  57      1948  11.26152      15
## 10 1701       9  58      1948  11.28407      15
## 13 2301       7  55      1946  11.01356      18
```

We'll take `lifeSat` as the response. Covariates include

- `age`: age at the time of the survey;
- `birthyear`: year of birth;
- `logincome`: log pre-tax income;
- `yearsEd`: years of formal education.

Types of variables

Macro variables: Vary across indviduals but not time - `birthyear,yearsEd`

Micro variables: Vary within individuals/across time - `age,logincome`.

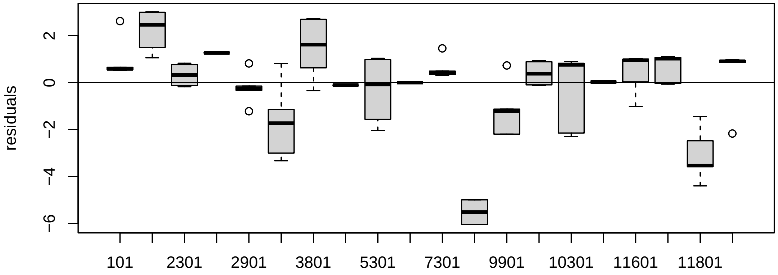
The types effects we may want to include are

- fixed effects for macro and micro variables, and their interactions.
- random effects for micro variables, and their interactions.

Random effects model with indpendent errors and main effects

```
fit0<-lm(lifeSat ~ age + birthyear + logincome + yearsEd, data=mappy )

summary(fit0)$coef

##                  Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept) 58.97223503 8.772117828  6.722691 1.989586e-11
## age          0.03301564 0.015410178  2.142456 3.220633e-02
## birthyear   -0.02845157 0.004463747 -6.373920 2.013280e-10
## logincome    0.09934413 0.009347222 10.628198 4.255265e-26
## yearsEd      0.03166438 0.010570551  2.995527 2.753540e-03
```

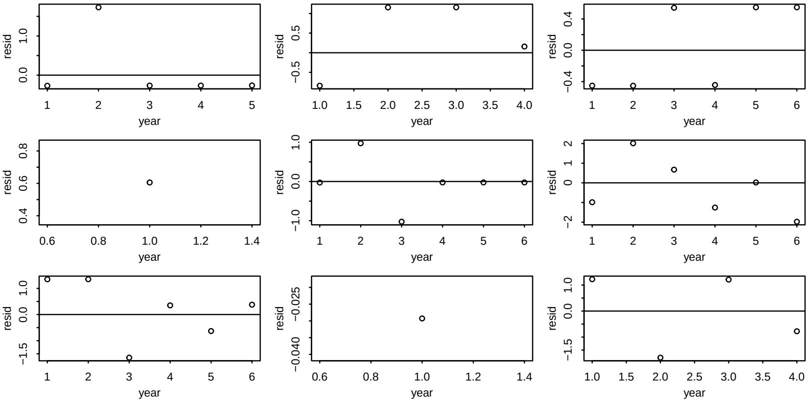
Residual plots:



Random effects model with indpendent errors and main effects

```
fit1<-lme(lifeSat ~ age + birthyear + logincome + yearsEd,
                    random= ~1|id,data=mappy , method="ML")

fit2<-lme(lifeSat ~ age + birthyear + logincome + yearsEd,
                    random= ~1+age|id,data=mappy , method="ML")

fit3<-lme(lifeSat ~ age + birthyear + logincome + yearsEd,
                    random= ~1+age+logincome|id,data=mappy , method="ML")
```

```
BIC(fit0)

## [1] 19471.85

BIC(fit1)

## [1] 17602.67

BIC(fit2)

## [1] 17619.65

BIC(fit3)

## [1] 17625.13
```


resid
year

AR1 covariance model

```
fit4<-lme(lifeSat ~ age + birthyear + logincome + yearsEd,
                    random= ~1|id,correlation=corAR1(),data=mappy,method="ML")

BIC(fit4)

## [1] 17590.9
```

Looking for interactions

```
add1(fit4,"age*birthyear",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##               Df   AIC    LRT  Pr(>Chi)
## <none>           17539
## age*birthyear  1 17528 12.859 0.0003359 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
add1(fit4,"age*logincome",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##               Df   AIC   LRT Pr(>Chi)
## <none>           17539
## age*logincome  1 17538 2.544   0.1107
```

```
add1(fit4,"age*yearsEd",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##             Df   AIC    LRT Pr(>Chi)
```

More interactions

```
add1(fit4,"birthyear*logincome",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##                     Df   AIC    LRT Pr(>Chi)
## <none>                 17539
## birthyear*logincome  1 17540 1.0472   0.3061
```

```
add1(fit4,"birthyear*yearsEd",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##                   Df   AIC    LRT Pr(>Chi)
## <none>               17539
## birthyear*yearsEd  1 17541 0.3618   0.5475
```

```
add1(fit4,"logincome*yearsEd",data=mappy,test="Chisq")

## Single term additions
##
## Model:
## lifeSat ~ age + birthyear + logincome + yearsEd
##                   Df   AIC     LRT Pr(>Chi)
## <none>               17539
## logincome*yearsEd  1 17540 0.51157   0.4745
```

“Final” model

```
fit5<-lme(lifeSat ~ age + birthyear + logincome + yearsEd + age*birthyear,
                    random= ~1|id,correlation=corAR1(),data=mappy,method="ML")

summary(fit5)

## Linear mixed-effects model fit by maximum likelihood
##   Data: mappy
##        AIC      BIC    logLik
##   17528.13 17586.53 -8755.065
##
## Random effects:
##  Formula: ~1 | id
##         (Intercept) Residual
## StdDev:    1.297903 1.238987
##
## Correlation Structure: AR(1)
##  Formula: ~1 | id
##  Parameter estimate(s):
##        Phi
## 0.09557113
## Fixed effects:  lifeSat ~ age + birthyear + logincome + yearsEd + age * birthyear
##                  Value Std.Error   DF   t-value p-value
## (Intercept)   907.3259 232.77704 3868  3.897832  0.0001
## age           -14.7224   4.10112 3868 -3.589834  0.0003
## birthyear      -0.4652   0.12016  988 -3.871912  0.0001
## logincome       0.0426   0.00983 3868  4.336077  0.0000
## yearsEd         0.0482   0.01833 3868  2.628823  0.0086
## age:birthyear   0.0076   0.00212 3868  3.590019  0.0003
##  Correlation:
##                (Intr) age    brthyr logncm yersEd
```